

ΑΝΑΛΥΣΙΣ ΠΕΡΙ ΤΑ ΣΧΗΜΑΤΑ

Restoring Aristotle's Lost Diagrams of the Syllogistic Figures

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οὐ γὰρ μόνον ἴσως δεῖ τὴν γένεσιν θεωρεῖν
τῶν συλλογισμῶν, ἀλλὰ καὶ τὴν δύναμιν ἔχειν τοῦ ποιεῖν.
APr 43 a 22

εἰ γὰρ τὴν τε γένεσιν τῶν συλλογισμῶν θεωροῖμεν καὶ τοῦ
εὐρίσκειν ἔχομεν δύναμιν, ἔτι δὲ τοὺς γεγενημένους ἀναλύοιμεν
εἰς τὰ προειρημένα σχήματα, τέλος ἂν ἔχοι ἡ ἐξ ἀρχῆς πρόθεσις.
APr 47 a 2

The above heading and *mottos* refer to Aristotle's most important and revealing issue, i.e., "the *analysis* which concerns the figures" (*APo* II 5, 91 b 13). The Stagirite consistently appeals not only to his most original treatise *Analytica*, but also to its crucial project and subject matter. Our attention will focus here on this significant finding made in the *Prior Analytics* that so far has not yet been sufficiently treated. We attempt to understand "the *analysis* concerning the figures" strictly in its own formulation, and, thus, to recapture, at

least to some extent, Aristotle's probable diagrams, which regrettably are missing from the extant text of the *Prior Analytics*. Unfortunately, only few scholars (e.g. Einarsen 1936; Tredennick 1938; Ross 1949; Kneale 1962; Rose 1967; Geach 1987; Englebretsen 1992, 1998) have suggested that Aristotle employed diagrammatic tables in teaching his analytics. We have earlier sought to reconstruct Aristotle's lost diagrams of the syllogistic figures and to show how his analytics and apodeictics should be regarded as a treaty about a heuristic analysis that aims to find the terms and premises for the syllogisms as demonstrations or scientific explanations (see the bibliography).

Aristotle's most famous logical theory is traditionally called the syllogistic, even though the philosopher himself did not use this notion. With reference to his inferential and demonstrative framework, Aristotle proposed the title and the project matter: *ta analytika*. Throughout centuries, many scholars of Aristotle have paid little attention to the significance of his analytical approach. Finally, has it been aptly stated that Aristotle transformed the analytical method employed in finding solution to geometrical theorems so as to develop a method of approaching problems in demonstration.¹ This is an original exposition of Aristotle analytics, which corroborates entirely our interpretation with reference to the analysis applied in Greek geometry. Nonetheless, an attempt to reconstruct the syllogistic figures was not undertaken by this author.

1. ἀναλαμβάνειν ἐν γραμμαῖς δύνάμιν εὐρετικὴν...
– δύνάμιν τινα εὐρεῖν συλλογιστικὴν...

At the conclusion of the *Sophistical Refutations* (34), Aristotle informs us that on the syllogistic reasoning (περὶ τοῦ συλλογίζεσθαι) “we had absolutely nothing earlier to mention, but we spent much time in experimental research (τριβῆ ζητοῦντες — *exercitatione quaerentes*)”. Accordingly, his initial and original project was „to discover a certain syllogistic ability about a given problem from the most endoxical (acceptable) belongings (δύνάμιν τινα εὐρεῖν συλλογιστικὴν περὶ τοῦ προβληθέντος ἐκ τῶν ὑπαρχόντων ὡς ἐνδοξοτάτων); for this is the function of dialectic in itself and of peirastic” (*SE* 183 a 37). Presumably, his laborious research was not restricted only to the heuristic *dynamis* for syllogising in the dialectic (topics), but was mostly concerned with the relevant heuristic *dynamis* for syllogising in the analytics, as we read in the course of the *Prior Analytics* (I 27-32)².

¹ Fifteen years ago, Patrick H. Byrne in his stimulating book *Analysis and Science in Aristotle* offered a new interpretation of the *Analytics* as a unified whole. Thus, Byrne (1997: XVII) showed “how influences from Greek geometrical analysis can be found in Aristotle's conception of a more general method of analysis, and how his remarks concerning both syllogism and demonstrative science can be interpreted within this larger context”.

² We cannot agree with a contrary view of L.A. Dorion who claims that in the *SE* 34 “Aristote s'y enorgueillit d'avoir découvert la dialectique et non pas la logique” (Dorion 2002: 215). But was Aristotle so proud only of having discovered the dialectic? Was he not aware of his painstaking and most fundamental ‘analysis concerning the figures’? The point is that the syllogistic *dynamis* is in dialectic as well in analytics. Note the analogical

“For surely one ought not only discern (θεωρεῖν) the genesis of syllogisms, but also have the ability to produce them (τὴν δύναμιν ἔχειν τοῦ ποιεῖν)” (*APr* 43 a 22).

“After this we should explain how we will reduce syllogisms to the aforementioned figures, for this part of our inquiry still remains to be done. For if we should discern the genesis³ of syllogisms and have the ability to find them (καὶ τοῦ εὐρίσκειν ἔχοιμεν δύναμιν), and then also analyze existing syllogisms into the aforementioned figures, our initial project should have come to the end.” (*APr*, 46 b 40-47 a 6; Striker's translation slightly modified).

In this respect, Aristotle's crucial project in the *Prior Analytics* was: 1) to discern (survey) the *genesis* of syllogisms, i.e., from which elements (terms and premises) they come about; 2) to work out an ability to find them, based properly on the syllogistic figures that show their *genesis*; and 3) to analyze or reduce (ἀναλύειν, ἀνάγειν) the produced ones into the figures previously stated (cf. *APr* I 32, 47 a 2-6; I 26-27, 43 a 16-24). These three research tasks are complementary and define the overall objective of the *Prior Analytics*.

At this point, we must note an often overlooked analogy between Aristotle's project of finding a syllogistic ability and the general purpose of an analysis in Greek geometry as a special resource which allows „to obtain in lines a heuristic ability for solving problems proposed (ἀναλαμβάνειν ἐν γραμμαῖς δύναμιν εὐρετικὴν τῶν προτεινομένων αὐτοῖς προβλημάτων” — Pappus, *Coll. Math.* 634, 5). In all probability, Pappus of Alexandria reported here the older Greek view on the purpose of geometrical analysis. In connection with this, let it suffice to quote the earlier and principal evidence for the definitions of analysis and synthesis, from the *scholium* on Euclid's Book XIII:

Ἀνάλυσις μὲν οὖν ἐστὶ λῆψις τοῦ ζητουμένου ὡς ὁμολογουμένου διὰ τῶν ἀκολουθῶν ἐπὶ τι ἀληθὲς ὁμολογούμενον. Σύνθεσις δὲ λῆψις τοῦ μολογούμενου διὰ τῶν ἀκολουθῶν ἐπὶ τι ἀληθὲς ὁμολογούμενον (Eucl. IV 198 Stamatis).

„Analysis est adsertio eius, quod quaeritur, ut concessi, qua per consequentias ad aliquid pervenitur, quod verum esse conceditur. Synthesis est adserio concessi, qua per consequentias ad aliquid pervenitur, quod verum esse conceditur” (Heiberg).

„Now analysis is the assumption of what is sought, as if it were admitted, through its consequences, up to something admitted as true. And synthesis is the assumption of what is admitted, through its consequences, up to something as true.”

Later Pappus (*Coll. Math.* 634, 11) summarized these procedures in the following way: ἀνάλυσις τοίνυν ἐστὶν ὁδὸς ἀπὸ τοῦ ζητουμένου ὡς ὁμολογουμένου διὰ τῶν ἐξῆς ἀκολουθῶν ἐπὶ τι ὁμολογούμενον συνθέσει· („Now analysis is a method from what is sought, as if it were admitted, through its successive consequences, up to something admitted in synthesis”).

definition of the syllogism in both cases, and the analogical initial research project in the *SE* 183 a 34 (περὶ τῆς ἐξ ἀρχῆς προθέσεως) and in the *APr* 47 a 5 (ἡ ἐξ ἀρχῆς πρόθεσις).

³ Here, the word *genesis* is transliterated. See below for Aristotle's claim that “what is last in the analysis is first in the genesis” (*EN* 1112 b 23).

These very terse definitions of the analysis-synthesis raise some complex questions,⁴ but without going into details and for the sake of greater clarity we can plainly show their interconnection on the synoptic diagram, where the opposing arrows will mark the convertible relation of both procedures.

a		<i>τὸ ὁμολογούμενον</i>		s
n	↑	<i>quod conceditur</i>		y
a			↓	n
l	↑	<i>διὰ τῶν ἀκολουθῶν</i>		th
y		<i>per consequentias</i>	↓	e
s	↑			s
i		<i>τὸ ζητούμενον</i>	↓	i
s		<i>quod quaeritur</i>		s

The arrow stands for the literal sense of *ana-lysis* as ‘regressive’ or ‘elevating’. Its direction is conceived as being upward, from what is sought through its antecedents to something admitted as true. Thereby, we emphasize *analysis* as a regressive upwards inference, and *synthesis* as a progressive downwards one. What is of paramount importance is that both procedures presuppose a relevant symmetry and reversibility between them.

It seems that the method of analysis exerted an influence on Plato’s *agrapha dogmata*,⁵ but was particularly adapted by Aristotle in his foundation of syllogistic and demonstrative knowledge. Indeed, the Stagirite was guided by analysis and synthesis, a combined method, already recognized by Plato who „used to put this *aporia* and inquired whether we proceed *from or toward the principles*” (*EN* 1095 a 35). Aristotle, being a great logician, was also perfectly aware of the fact that to make an analysis is not easy and it depends on the validity of the inferential conclusion-premise conversion, as it is characteristic of mathematics.

„If it were impossible from falsehood to show truth, it would be easy to analyze (*τὸ ἀναλύειν*); for then [the analysis] would convert from necessity. Let *A* be true; and if this is true, *these* things, which I know to be true — e.g. *B*. Then from the latter I will show that the former is true. Now [arguments] in mathematics convert more often, because they assume nothing accidental, and in this they differ from dialectical arguments, but rather definitions” (*APo* I 12, 78 a 6-13).

Undoubtedly, Aristotle had such a correlation of analysis-synthesis in mind which he incorporated into his crucial project of the *Prior Analytics*, as we have already mentioned. In order to determine what exactly the philosopher meant by *analysis* and *analyein* with relation to the syllogistic figures, we must firstly quote his key passages referring to the analyzing diagrams.

⁴ Different translations (Hintikka-Remes 1974; Knorr 1986). But for a good clarification see especially Berti 1984 and Byrne 1997.

⁵ For our interpretation of Plato’s ‘analytic system of principles’, see Wesoły 2012.

2. ἀναλύειν διαγράμματα ...

Presumably, Aristotle's original inspiration for his analytics came from the geometrical method of finding solutions to what is sought-for-lines by means of drawing diagrams. It seems that an *analysis* conceived in this way was primarily connected with *diagrammata*. Thus, the very notion of διάγραμμα (*lit.* 'lines through', from *diagraphein* 'to mark out by lines,' 'draw a line between,' 'delineate'), i.e., a figure marked out by lines, derives from geometric terminology and means a plane figure (σχήμα ἐπίπεδον) in which the points and lines are appropriately arranged. Drafting lettered diagram was used for didactic reasons, thereby, facilitating the understanding and providing the solution for various complex issues (cf. διδασκαλίας χάριν — *Coel.* 280 a 1). Similarly, σχῆμα (from ἔχειν, σχεῖν 'to have, hold, be in such position') means a 'drawing' or 'sketch'; hence a 'form', 'shape', or 'geometrical figure', employed for making something clearer or easier to understand. Thus, in the procedure of drawing these diagrams-figures, it has become possible to invent relevant geometrical theorems and proofs. For this reason *diagrammata* were regarded by the Greeks as metonyms of geometrical theorems.⁶

There are at least three passages in which Aristotle expounds his approach for 'analyzing the *diagrammata*' (a transliteration of the Greek term seems more convenient than the inaccurate translation 'geometrical proofs'):

„For the man who deliberates seems to inquire and analyze (ζητεῖν καὶ ἀναλύειν) in the way described as in the case of a *diagramma* (ὥσπερ διάγραμμα) [...], and what is last in the *analysis* is first in the *genesis*.” (*EN* III 3, 1112 b 20-25).

“Sometimes this also happens in the *diagrammata*; for having analyzed, we sometimes cannot synthesize again” (*SE* 175 a 31).

Such a geometrical analysis consists in actual dividing a geometrical figure as we read in following passage:

“The *diagrammata* are discovered in actuality; for we discover them by dividing. If they had been divided they would have been evident; but as it is they are in there potentially.” (*Metaph.* 1051 a 23).

In general, it should be noted that a demand for cognitive perception or visualization is very characteristic of Aristotle's analytical approach. In the *De Anima* (III 8, 432 a 7) the philosopher makes his view explicit by claiming that images are indispensable for thinking, for there is no learning or understanding without perceiving, „and whenever one surveys (θεωρεῖν) one must simultaneously survey with an image”. At the beginning of the *On Memory* (449 b 31), he confirms his conviction: „thinking (νοεῖν) is not possible without an image; for the same affection occurs in thinking as in drawing a diagram” (ἐν τῷ νοεῖν ὅπερ καὶ ἐν τῷ διαγράφειν) (sic!).

Therefore, we can ascertain that Aristotle was well aware of the sense ἀναλύειν διάγραμμα which might mean primarily to inquire a figure by dividing it into its simplest elements (points, lines, angles, ratios), and consequently to discover some constructed

⁶ Somewhat differently R. Netz, 1999: 35–43.

interconnections which reflect its ostensive, intelligible and demonstrative framework. Indeed, on such diagrammatic analysis — as we shall see — Aristotle based his three figures of syllogisms.

3. τὰ τῶν διαγραμμάτων στοιχεῖα – τὰ τῶν ἀποδείξεων στοιχεῖα

At any rate, Aristotle's recognition of the geometrical 'analyzing *diagrammata*' enabled him to formulate the lettered *schemata* for syllogisms in a perceptive and reductionist pattern. It must be emphasized that the Stagirite conceived the elements of diagrams and of demonstrations in a similar manner. In his analytic (reductionist) approach, the *stoicheia* (elements, letters) are prior in order with respect to the *diagrammata* — *schemata* (*Cat.* 14 a 39), and such *stoicheia* are the constituents of demonstrations. The following passages are of paramount importance for our diagrammatic interpretation of Aristotle's lost syllogistic figures.

καὶ τῶν διαγραμμάτων ταῦτα στοιχεῖα λέγομεν ὧν αἱ ἀποδείξεις ἐνυπάρχουσιν ἐν ταῖς τῶν ἄλλων ἀποδείξεσιν ἢ πάντων ἢ τῶν πλείστων (*Metaph.* 998 a 25-26).

Et figurarum ea dicimus elementa quorum demonstrationes in aliorum aut omnium aut plurium demonstrationibus insunt (Bekker, III, 1831: 489).

"And we speak of the elements of *diagrammata*, the demonstrations of which are present in the demonstrations of the others, either of all or of most."

παραπλησίως δὲ καὶ τὰ τῶν διαγραμμάτων στοιχεῖα λέγεται, καὶ ὅλως τὰ τῶν ἀποδείξεων· αἱ γὰρ πρῶται ἀποδείξεις καὶ ἐν πλείοσιν ἀποδείξεσιν ἐνυπάρχουσαι, αὗται στοιχεῖα τῶν ἀποδείξεων λέγονται· εἰσὶ δὲ τοιοῦτοι συλλογισμοὶ οἱ πρῶτοι ἐκ τῶν τριῶν δι' ἐνὸς μέσου (*Metaph.* 1014 a 32-b 3).

Similiter autem figurarum quoque elementa dicuntur, ac similiter demonstrationum. Primae enim demonstrationes, quae in pluribus demonstrationibus insunt, hae elementa demonstrationum dicuntur. Sunt autem tales primi ex tribus per unum medium syllogismi (Bekker, III, 1831: 496).

"In much the same way, the elements of *diagrammata* are called, and in general the demonstrations. For the primary demonstrations that are present in many demonstrations, are called the elements of demonstrations; these are the primary syllogisms consisting of three [terms] through one middle."

This is surely a remarkable reference to the diagrammatic elements (= terms) of the primary syllogisms of the first figure consisting of three terms through one middle term, whereas other two figures are included in this first figure.⁷ The inventive graphical arrangement of the three terms for the three syllogistic figures — as in our interpretative proposal — will be explained later, but it should be emphasized here that these *schemata*

⁷ For the meaning of τοιοῦτοι συλλογισμοὶ οἱ πρῶτοι ἐκ τῶν τριῶν δι' ἐνὸς μέσου we accept the exegesis *ad locum* of Alexander (365, 22) and of Asclepius (308, 2).

were primarily modeled on some *diagrammata*. Hence, the invention of the syllogistic figures can be seen as an extension of the diagrammatic analysis.

At this point, we must yet emphasize that for Aristotle the elements (στοιχεῖα) of demonstration are the proper terms (ὄροι) that construct propositions which constitute the immediate propositions. „And the elements are as many as terms; for the propositions containing these terms are the principles of the demonstration” (*APo* I 23, 84 b 27).

In the *Prior Analytics* I 30, there is yet another remarkable reference to the diagrammatic notation of the three syllogistic terms. Aristotle gives a piece of methodological advice on how one must by trial and error discern the three terms of ὑπάρχειν in order to hunt for the syllogistic premises. This very useful instruction concerns the method common to all subjects of researches.

“For one must discern both terms that belong to them and that they belong to, and be supplied with as many of those terms as possible. One must examine them through the three terms (διὰ τῶν τριῶν ὄρων σκοπεῖν), in one way when refuting, in another way when establishing something; and when it is a question of truth, for the terms that have been *diagrammed* to belong truly (κατὰ μὲν ἀλήθειαν ἐκ τῶν κατ' ἀλήθειαν διαγεγραμμένων ὑπάρχειν), for dialectical syllogisms from premises according to opinion.” (*APr* 46 a 5-10 — Striker's translation modified).

Aristotle refers here *expressis verbis* to the three syllogistic terms belonging as *diagrammed* in accordance with the truth.⁸ This passage and others will be integrated into a possible recapturing of Aristotle's lost diagrammed and lettered analytical figures.

Furthermore, it is worth emphasizing that the Greek τὰ στοιχεῖα stands also for the letters, and Aristotle used letters or lettered diagrams to illustrate and analyze complex issues (Bonitz, *Index Ar.* 178 a 2-25). His mode of employing the letters in the analytics does not seem that different from the way in which letters were used in geometrical diagram-figures, theorems and proofs. Indeed, such letters would make sense if they referred to some diagrammatic configuration. From this point of view, such letters should be considered rather as placeholders than as logical variables. We cannot, however, discuss this matter here.⁹

It follows from the above quotations that the analysis concerned primarily a heuristic inquiry through constructible diagrams-figures. For Aristotle, to ‘analyze’ is to make an investigation of how to discover terms and premises from which to deduce the desired conclusion, or of how to resolve a conclusion into its terms constructing premises. Thus, the Greek geometrical analysis and Aristotle's analytics constitute a heuristic or regressive procedure: from a given *problema* (conclusion) to grasp, by means of diagrams, the

⁸ Cf. *Hist. anim.* 566 a 15 ἐκ τῶν ἐν ταῖς ἀνατομαῖς διαγεγραμμένων — a reference to the diagrams in the Anatomies, *Rhet.* 1378 a 28: ἐπὶ τῶν προειρημένων διεγράψαμεν τὰς προτάσεις — Just as we have drawn up a list of propositions on the subject discussed.

⁹ For an important discussion see Ierodiakonou 2002: 127–152. We agree with G. Striker (2009: 86) that: “The letters do not appear in actual syllogisms, but only in proofs, as placeholders for concrete terms. Aristotle proves the validity of a form of argument by showing for an arbitrary case how a conclusion can be derived from premisses of a given form by elementary rules”.

relevant elements as terms and premises of the syllogism. Conversely, to this heuristic procedure, *sylogismos* constitute a *synthesis*, namely a deductive or progressive reasoning.¹⁰ Aristotle's account of both procedures constitutes a complementary and convertible order of inquiry, i.e., 1) the analytic, heuristic or regressive, and 2) the synthetic, genetic or progressive.¹¹

Much has been written on Aristotle's syllogistic as a deductive procedure,¹² but its analytical or heuristic strategy of finding terms and premises of syllogisms has generally been overlooked, mainly because the relevant diagrams of the three figures have not been adequately taken into account. Before trying to reconstruct them, let us first see Aristotle's analytical way of defining the syllogism and its basic elements: the three terms linking the predications.

4. συλλογισμὸς δὲ ἐστὶ λόγος ἐν ᾧ τεθέντων τινῶν [sc. ὄρων] ...

It is often claimed that Aristotle's syllogism was a deductive argument in which a conclusion is necessarily inferred from the two premises. But this is not quite true. As we shall shortly see, in his syntactical and predicative determination of the terms of the three figures, the philosopher did not use *expressis verbis* such notions as premises and conclusions.¹³ What he calls πρότασις (*literally* that which is put forward) does not mean a „premise” (as it is customarily rendered) but rather “a *logos* affirming or denying something of something” (*APr* 24a16), i.e., a categorical proposition. The verb προτείνω (“to put forward”) in relation to ὄροι means „to stretch” as of lines joining two points in a diagram. Hence, the synonym for πρότασις is here διάστημα (*APr* 35 a 31; 38 a 4; 42 b 10), on analogy with interval in the harmonic diagrams. Thus, the three ὄροι fit schematically together like intervals in a diagram that join one extreme to another through the middle. Similarly, what Aristotle calls συμπέρασμα (*literally* „termination”, “finishing”) does not sit quite well with our “conclusion”, as is evidenced by the verb (συμ)περαίνειν (accomplish jointly) that is used for completing a syllogism or proving a πρόβλημα (anything thrown forward or projected). Thus, Aristotle conceives the *problema* as *symperasma*, i.e. an object of inquiry, modeled on the geometric analysis of problems.¹⁴

¹⁰ Thus, Aristotle regards the production of a syllogism as a genesis from the „what-it-is” (*Metaph.* 1034 a 31-33).

¹¹ The reflection on both these methods is also to be found in *Metaph.* 1044 a 24: “For one thing comes from another in two senses: either progressively (πρὸ ὁδοῦ), or by resolving into its principle (ἀναλυθέντος εἰς τὴν ἀρχήν)”.

¹² For modern views, see especially Łukasiewicz, Patzig, Corcoran, Lear, Englebretsen, Mignucci, Barnes, Smith, Striker, Criveli (see the Bibliography).

¹³ Instead of these, he used such expressions as ὄροι, διάστημα, πρότασις, κατηγορεῖσθαι, τίθεσθαι, θέσις, τὸ μείζον, τὸ μέσον, τὸ ἔλαττον, τέλειον, σχήμα, etc. For a good discussion of this mathematical terminology see especially Einarson 1936.

¹⁴ This was well suggested by Lennox (1994: 73–76).

At the beginning of the *Prior Analytics*, Aristotle clarifies the syntax of predication (*resp.* ‘to belong’ one term to another). By the ‘term’ ὅρος (*literally* ‘limit’ or ‘limiting point’) Aristotle means “that into which a *protasis* is analyzed, namely what is predicated and what it is predicated of” (*APr* 24 b 18).

Similarly, Aristotle’s notion συλλογισμός does not correspond to the English “syllogism” (in Greek συλλογίζεσθαι is to ‘sum things up’, ‘to add up’, ‘to compute’). In the context of his analytical approach, it is extremely anachronistic and misleading to render this concept as ‘deduction’.¹⁵

Although the general notion of *syllogismos* in the analytic (*APr* I), dialectic (*Top.* I 1) and rhetoric (*Rhet.* I 2), seems to be defined quite identically as a valid deductive argument, there is, however, a great difference in their *protaseis*: demonstrative or dialectical, respectively. This difference corresponds to a further specification in analytical respect as well. If we consider Aristotle’s analytical approach, then we can adopt a more contextual and relevant reading of his definition of a ‘syllogism’.

συλλογισμός δέ ἐστι λόγος ἐν ᾧ τεθέντων τινῶν ἕτερόν τι τῶν κειμένων ἐξ ἀνάγκης συμβαίνει τῷ ταῦτα εἶναι. λέγω δὲ τῷ ταῦτα εἶναι τὸ διὰ ταῦτα συμβαίνειν, τὸ δὲ διὰ ταῦτα συμβαίνειν τὸ μηδενὸς ἕξωθεν ὄρου προσδεῖν πρὸς τὸ γενέσθαι τὸ ἀναγκαῖον.

τέλειον μὲν οὖν καλῶ συλλογισμὸν τὸν μηδενὸς ἄλλου προσδεόμενον παρὰ τὰ εἰλημμένα πρὸς τὸ φανῆναι τὸ ἀναγκαῖον, ἀτελεῆ δὲ τὸν προσδεόμενον ἢ ἐνὸς ἢ πλειόνων, ἃ ἔστι μὲν ἀναγκαῖα διὰ τῶν ὑποκειμένων ὄρων, οὐ μὴν εἴληπται διὰ προτάσεων (*APr* 24 b 18).

“A syllogism is a *logos* [formula, argument] in which, certain [terms] being posited, something other than what was laid follows of necessity because these [terms] being so. By ‘because these being so’, I mean ‘following through them’, and by ‘following through them’ I mean that no term is required outside for generating the necessity.”

I call a syllogism perfect if it requires no other [term] beyond these assumed for the necessity to be evident; and imperfect if it requires one or more [terms] that are necessary through the terms laid down, but have not been assumed through propositions.”

It is important to notice that in some crucial details, we read Aristotle’s definition of the syllogism quite differently. Thus, the indefinite pronoun in the expression τεθέντων τινῶν does not refer to „certain [things]” or to „certain [premises]”, as all translators assume, but rather to „certain [terms]” (τινῶν *scil.* ὄρων) being posited of necessity by predications (*sic!*).¹⁶

¹⁵ Thus, improperly Barnes (1996) and Smith (1989) in their translations. According to D. Keyt (2009: 36), “this is wrong. ‘Deduction’ is a syntactic, or proof theoretic, concept, whereas a reference to necessity in Aristotle’s definition would seem to indicate that he is defining a semantic, or theoretic, concept.” G. Striker (2009: 79) with some hesitation decided to keep the word ‘syllogism’ as a transliteration of the Greek instead of ‘deduction’. She justly states that “the translation ‘deduction’ includes too much: not every deduction is or can be used as an argument, and the conditions Aristotle spells out in his definition make sense only if one keeps in mind that what he sets out to define is the notion of valid deductive argument” (ibidem, p. 79).

¹⁶ See for instance: *APr* 29 b 37 τιθεμένων τῶν ὄρων — “when the terms are posited”; *APo* 56 b 37 τεθέντων οὖν ὄρων τοιοῦτων — “thus these terms being posited”; *APr* 47a36 μέσον δὲ θετέον τῶν ὄρων; 72 b 36 — δηλον ὅτι τοῦτο συμβαίνει τριῶν ὄρων τεθέντων. Note that τεθέντων ὄρων (aor. part. pass, from τίθημι — referring

There is good evidence to show that the terms (predicates — subjects) involved in the categorical predication play a fundamental role in the syllogistic. It can be seen as an extension of his predicate logic, namely of the relation between the subject-predicate terms¹⁷. In the course of the *Prior Analytics*, Aristotle “mentions propositions and terms almost in the same breath”¹⁸.

Hence, positing these terms (ὄροι) in some array makes it ‘follow of necessity’, i.e., makes the transition to other configurations of terms in predications according to the analytical *schemata*. Evidently, then, in the definition of a syllogism, a *logos* like ‘proportion’ is to be understood as a certain connection of the extremes with the middle terms. Strictly speaking, the notion of analytical syllogism makes sense only if we interpret it within the framework of the three figures, which implicitly refer to a syntactical position of terms in diagrams that are now lost, but that once accompanied the text of the *Prior Analytics*.

In the course of the *Analytics*, a proposition (πρότασις) was usually represented by two terms (symbolized by letters AB), and in this way Aristotle expressed the predication: ‘A is predicated of B’, or ‘A belongs to B’ (the predicate before the subject). It seems that it was from Aristotle’s invention of categorical predication that his concept of figured syllogism departed and was further developed. The question was surely how to connect a linear AB notation for a proposition with a two-dimensional ABΓ notation for a syllogism. If it comes to a plane figure (σχήμα) within a pivotal position of the middle, only three combinations with two extremes are possible. Now, we can anticipate what the syntactical and predicative term-order between three letters (ABΓ) will be. Before we display these in the diagrammed figures, let us first show simply in a linear array the three syllogistic patterns: 1. AB & BΓ → AΓ; 2. BA & BΓ → AΓ; 3. AB & ΓB → AΓ (this is how they are customarily presented, albeit our thesis has it that in Aristotle they were originally two-dimensional or planar).

In the analytical configuration, all syllogisms come from the terms (predicates and subjects) that complete two categorical *protaseis* in such a way that from certain terms (predications constructing premises) that are posited something other follows of necessity. Accordingly, in the syllogistic ‘follow of necessity’, the syntax consists in some predi-

to ὄροι), but εἰλημμένων προτάσεων (perf. part. pass, from λαμβάνω — referring to προτάσεις). Cf. *APr* 33 a 7; 33 a 15; 33 a 31; 34 a 5; 35 a 4; 35 a 14; 37 b 31; 8 b 33; 39 a 24; 39 b 1; 40 b 39; 47 a 28 — Cf. ὑποκειμένων ὄρων, οὔτως ἐχόντων τῶν ὄρων, τῶν ὄρων ὄντων, τῶν ὄρων ὄντων πρὸς τὸ μέσον. See Philoponus, *APr*, 13, 2, p. 323, 20 ἀναγκαῖον μὲν τι συμβαίνει ἐκ τῶν τεθέντων ὄρων, καὶ ὁ συλλογισμὸς δὲ ἀναγκαῖος.

¹⁷ According to D.W. Graham (1987: 41, 43): „Aristotle’s syllogistic is a logical system designed to account for the logical properties of subject-predicate propositions. [...] Aristotle is thinking of his syllogistic as a calculus for deducing predications from other predications”. Similarly J. Lear (1988: 221): “Aristotle is introducing a logic of predication. It is a study of what predicational relations follow from others”. See also A.T. Bäck (2005: 252sq.): “Even Aristotle’s main inferential vehicle, the syllogism, can be seen as an extension of an aspect theory of predication. Aristotle holds a syllogism to give an explanation why the predication made in the conclusion (‘S is P’) is true.”

¹⁸ Thus P. Crivelli (2012: 116). Cf. G. Striker (2009: 81): “Aristotle probably speaks of terms rather than premises being added ‘from outside’ in order to indicate that he is thinking of assumptions that are logically independent of the premises given.”

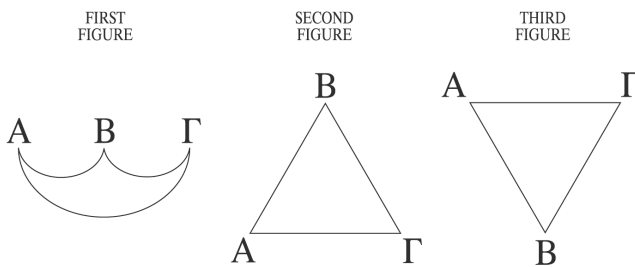
cational interconnections between the three terms involved in some schematized and completed arrangement.

It has been well recognized that Aristotle's syllogistic formula „following of necessity” is not based on implication, inference or the *modus ponendo ponens*, for the philosopher was in no way conscious of the modern notion of logical consequence in the semantic or syntactic sense that we know.¹⁹ In a very different manner, Aristotle considers following of necessity within the perspicuity of predicative transitivity on the basis of the middle term in the first figure to which the moods of other two figures are reduced.

A disregard for Aristotle's analytical approach was already in antiquity due to the fact the Stoics, who criticized his syllogistic, did not base the entailment on the connection of terms-predicates in the figured arguments, but rather on the conditional (τὸ συνημμένον) formed from a combination of the premises as antecedent and the conclusion as consequent (Sex. Emp. *PH* II 134–43). They used, however, *analysis* as a reduction of arguments to the indemonstrable and this *analysis* was performed by using the four *themata* (cf. Diog. Laert. VII 78; Sex. Emp. *Math.* VIII 228–240).²⁰

5. τὰ τρία σχήματα τῶν συλλογισμῶν - ἀναστάσεως πείρα

In some late Greek commentaries on Aristotle's *Analytica*, especially in several scholia to Ammonius (*in Apr* VIII 20–25; X 10 — XI 2; 39, 9) and Philoponus (*in Apr* 65, 20–23; 87, 8), there have been preserved certain lune and triangle diagrams for the three syllogistic figures.²¹ These appeared, with their Greek lettered notation, more or less in such a manner:



Subsequently, such diagrams were transferred to various Byzantine and medieval Latin manuscripts and were common in Italian Renaissance (e.g. *Compendium de regulis et formis ratiocinandi* by John Argyropoulos and even *De Progressu Logicae Venationis* by

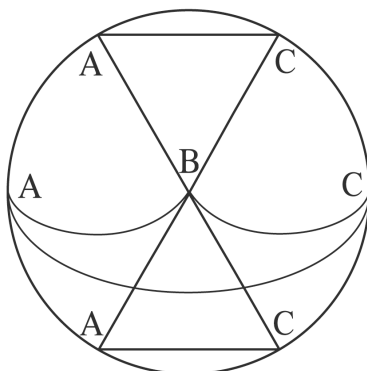
¹⁹ For this point, see Lear 1980.

²⁰ For a new reconstruction of the stoic syllogistic see Bobzien 1996.

²¹ Here, we are indebted to Rose 1968: 133–136. For illustrations, see Cacouros, 1996, 102–103.

Giordano Bruno). Consequently, these three figures were ingeniously combined into a single *figura complicationis*, which is reflected in the following diagram:

FIGURA COMPLICATIONIS TRIUM FIGURARUM

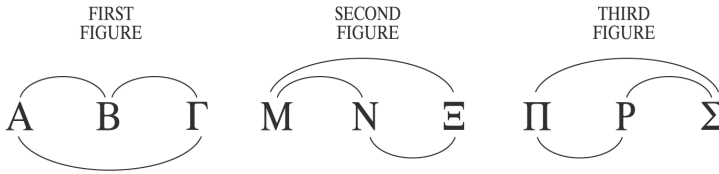


This was an attempt to expound *figurae et modi retiocinationis* in a visual and unified manner. However, in the traditional logic somewhat different notations were adopted: no longer three, but four syllogistic figures. Evidently, this goes beyond Aristotle's original threefold formulation, but the main deviation lies in the mode and order of predication (the subject before the predicate), as we read in many textbooks and standard representations of these four figures, namely in vertical columns that show the major and minor premise, and separately the conclusion; the symbols being: M — *terminus medius*, P — *praedicatum*, S — *subiectum* (below with the subject first).

FIRST FIGURE	SECOND FIGURE	THIRD FIGURE	FOURTH FIGURE
M - P	P - M	M - P	P - M
<u>S - M</u>	<u>S - M</u>	<u>M - S</u>	<u>M - S</u>
S - P	S - P	S - P	S - P

Needless to say, such a notation was hardly Aristotelian and it, therefore, led to some further distortions of his analytical prototype of syllogistic. As we shall see, the introduction of the fourth figure indicates a serious deviation from Aristotle's original threefold disposition of the middle term in respect to the both extremes.

Hence, W. and M. Kneales (1962: 72) sought justly to avoid the inconsistency by suggesting the following schemes of only three possible figures, which are similar to some diagrams to be found in manuscripts of Ammonius' *Commentary to the Prior Analytics*. Here, the syllogism is expressed as a linear array of three terms with the use of curved lines to connect the terms of a proposition (placing the premises-links above the letters and the conclusion-link below the letters)²²:



In the above proposal, as we can see, there are three different letter specifications for each figure. Although in the *APr* I 4–6, Aristotle initially used these various triads of letters, he, nevertheless, afterwards viewed the issue of analyzing syllogisms in a unified matter by means of the first three letters of Greek alphabet (ΑΒΓ). This is how they appear in ancient commentaries and scholia and that is why for the sake of clarity we use the first three letters in our reconstruction of Aristotle's diagrams.

The exact way in which Aristotle defined these figures seems *prima facie* unclear and has perplexed many scholars. However, a careful reading of the *Prior Analytics* (I 4–6 and elsewhere) helps to reconstruct Aristotle's diagrammed figures in a manner consistent with their original framework.



Having established the elements (στοιχεῖα) from which all analytical syllogisms come about, Aristotle invented the three figures within the pivotal function of the middle term, modeled evidently — as we can see — on some geometrical or harmonic diagrams which have not been preserved. Such diagrams were used to express systems of intervals and concords by means of lines, proportions and numbers.²³ Aristotle used the same notions

²² Such an attempt to render them as a linear array of three terms was also discussed and modified by L.E. Rose (1968: 16–26; 133–136).

²³ Cf. *Metaph.* 1078 a 14. Aristoxenus in his *Elements of Harmonics* (6.10.12; 12.15; 36.2; 42.14 Da Rios) refers to the harmonikoi whose “diagrams displayed melodic order in its entirety”, some of them seek to compress the diagram (κατασκευάσαι βουλομένοις τὸ διάγραμμα). Most surprisingly, Aristotle uses this verb κατασκευοῦται with reference to the procedure of ‘compression’ displayed by analytical figures (*APo* II 14, 79 a 30; II 23, 84 b 35). In the light of available sources (Philolaus, B 6; Hippocr. I 8; Aristot. fr. 908 Gigon; *Probl.* XIX 7; 25; 32; 47), we have sought to reconstruct the diagram of the enharmonic heptachord (seven string scale), in which the extreme terms (ὑπάτη — νήτη) occur in conformity with the constant intervals and dependent on the middle (μέση). Cf. Wesoly (1990: 91).

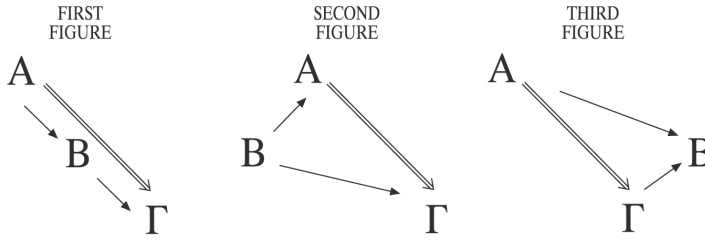
of interval (διάστημα) as a synonymous of *protasis*, and, when doing so, he was probably inspired by some two-dimensional harmonic diagrams within the pivotal function of the middle term linking the two extremes.²⁴

In accord with its name, *schema* — as plane figure — assumes no linear (rectilinear or curved), but rather planar or two-dimensional arrangement of terms. We must, therefore, take into account both a vertical (ἄνωθεν — κάτωθεν) and a horizontal (πρῶτον — ἔσχατον) order and the position of the three terms involved.²⁵ The extremes (τὰ ἄκρα) are arranged in such a relatively vertical order that the major (τὸ μείζον) lies above and precedes the minor (τὸ ἔλαττον), as their names indicate. Between them, the middle (τὸ μέσον) occurs vertically, while its horizontal position varies so that in the first figure it lies inside (εἴσω), but in the second and the third is outside of (ἔξω) the extremes. Only in such a two-dimensional array of the three terms involved, can we clearly understand Aristotle's striking claim that in the second figure the major extreme lies nearer to the middle and the minor is further from it, whereas in the third figure, conversely, the major is further from the middle, and the minor is nearer to it. Hence, the pivotal place of the middle: firstly, inside and between the extremes; secondly, outside them and first in position; and thirdly, outside, too, but last in position.

For greater clarity, we propose a reconstruction of the diagrammed figures in three successive steps. Firstly, let us display the three terms-letters (as Aristotle often does): *A* — the major; *B* — the middle; *Γ* — the minor. In Aristotle, the predicate is before the subject, so we show the direction of predication (belonging) in the premises, from left to right, by relatively vertical arrows,  while in the conclusion the term-order of predication remains fixed *AΓ* . But in Aristotle's definitions of the three figures, there is no mention of a separated conclusion (συμπέρασμα) which seems to be a part of the three-terms notation and to appear alongside within the two premises (διαστήματα). In the diagrams, its formula is the same, for it is the *sylogismos* of the extremes (see below the *AΓ* configuration). Of course, our use of the arrows serves here only as an interpretive indicator of Aristotle's order and position of the three syllogistic terms involved (evidentially then the major term in the second figure next to the middle, while in the third figure further from it).

²⁴ The very notion of the terms (horoi) — two extremes and one middle — originates presumably from the Pythagorean theory of proportion with the arithmetic, geometric and harmonic means (cf. Archytas, fr. B 2 DK; *Epinomis* 991 a-b).

²⁵ Aristotle's expressions in *APr* (I 25) and *APo* (I 32) are very instructive. They refer to the arrangement of terms involved in the figures. Thus: ἢ γὰρ ἔξωθεν ἢ εἰς τὸ μέσον τεθήσεται ὁ παρεμπύπτων ὄρος (*APr* I 25, 42 b 9), i.e., — „for the extra term will be added either from outsider or in the middle”; and: ἀνάγκη δέ γε ἢ εἰς μέσα ἀρμόττειν ἢ ἄνωθεν ἢ κάτωθεν, ἢ τοὺς μὲν εἴσω ἔχειν τοὺς δ' ἔξω τῶν ὄρων (*APo* I 32, 88 a 34-36), i.e., „it is necessary to fit (attach) either into the middle [terms] or from above or from below, or else to have some of their terms inside and others outside”. It is in fact hard to interpret these words in a way other than via an allusion to the syntactical and dimensional positions of terms involved in the diagrammed figures. Unfortunately, the commentators do not mention this. See Barnes 1994: 195-196 and Mignucci 2007: 242.



Up to this point, we can be relatively certain as to the diagrammatic and syntactic arrangement of the three terms involved in the three syllogistic figures. Aristotle in his definitions of these figures refers principally to the order and position of terms in some lost diagrams. The three-term notation for the syllogism is expressed in three figures, which “we may recognize by the position of the middle” (*APr* 47 b 13). Evidently, in such a combination of the middle term there is no place for the so-called fourth figure, the introduction of which indicates a certain misunderstanding of Aristotle’s original notation.

In the three terms notation, there occur some predicative relations that link the two *protaseis* (*diastemata*) with one *symperasma*. Hence, in our reconstructive diagrams-figures we should also display the relevant predicative patterns. However, we do not know how Aristotle marked the predicating expressions with negation, quantifiers and modalities, when in his schematized figures he used the lettered diagrams. Although there is no textual evidence here, we can — in our second step of reconstruction — display them by the four symbols known from medieval logic as constants: *a*, *e*, *i*, *o* (see below alongside the arrows).²⁶

²⁶ In this respect, there are some most striking similarities with Psellus’ notation in his *De tribus figuris* and with a textbook of logics written by John Chortasmenos (see M. Cacouras 1996: 99-106):

Ἐφοδος σύντομος καὶ σαφὴς τῆς εὐρέσεως τῶν συλλογισμῶν τῶν τριῶν σχημάτων τῆς λογικῆς πραγματείας τοῦ Ἀριστοτέλους γεγονυία παρὰ τοῦ ὑπάτου τῶν φιλοσόφων καὶ πατρικίου κυροῦ Μιχαὴλ τοῦ Ψελλοῦ, πῶς ὀφείλει εὐρίσκειν ὁ ζητῶν ἕκαστον αὐτῶν ὡς ἔχει τάξεως, ἥτοι ἐν πρώτῳ ἢ ἐν δευτέρῳ ἢ ἐν τρίτῳ σχήματι.

Ἰστέον οὖν ὅτι τὸ μὲν α ἐν ἑκάστῳ στιχιδίῳ ἀντὶ τοῦ ‘πᾶς’ προσδιορισμοῦ λαμβάνεται, τὸ δὲ ε ἀντὶ τοῦ ‘οὐδεὶς’, τὸ δὲ ἰῶτα ἀντὶ τοῦ ‘τίς’, τὸ δὲ ο ἀντὶ τοῦ ‘οὐ πᾶς’. λαμβάνονται δὲ οἱ μὲν δύο προσδιορισμοὶ οἷοι εἰσὶν ἐπὶ τῶν προτάσεων, ὁ δὲ τρίτος ἐπὶ τοῦ συμπεράσματος.

α^{ον} σχήμα, α^{ος} τρόπος † γράμματα. β^{ος} ἔγραψε. γ^{ος} γραφίδι. δ^{ος} τεχνικός.

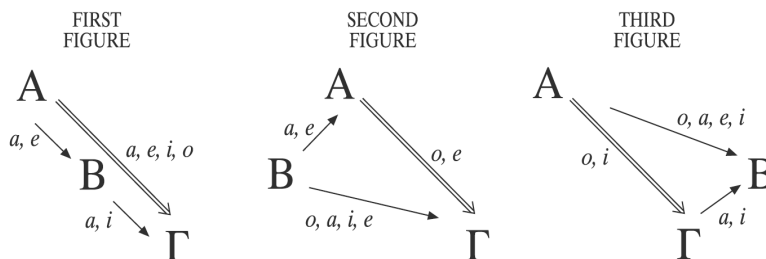
β^{ον} σχήμα, α^{ος} τρόπος † ἔγραψε. β^{ος} κάτεχε. γ^{ος} μέτριον. δ^{ος} ἄχολον.

γ^{ον} σχήμα, α^{ος} τρόπος † ἄπασι. β^{ος} σθεναρός. γ^{ος} ἰσάκις. δ^{ος} ἀσιπιδι. ε^{ος} ὀμαλός. ζ^{ος} φέριστος.

« Méthode brève et claire pour savoir à quelle figure, parmi les trois que mentionne Aristote dans sa *Logique*, correspond chaque syllogisme ; conçue par Sire Michel Psellos, hypatos ton philosophôn et patrice, cette méthode indique la manière dont on doit procéder afin de trouver l’ordre de chaque syllogisme, c’est-à-dire s’il a été fait suivant la première figure, la seconde ou la troisième.

Il faut savoir que [la lettre] alpha dans chaque ligne est prise au lieu de Chacun ; ainsi, celle-ci [scil. cette lettre] exprime l’universel. La lettre epsilon [est prise au lieu de] Aucun, la lettre iôta [est prise au lieu de] Quelqu’un et la lettre omi- cron [au lieu de] Non pas chacun. Les deux [premières] indications notées [scil. dans les schémas et figures syllogistiques qui suivent] [servent à désigner] les prémisses, et la troisième [sert à désigner] la conclusion.

Première figure, premier mode concluant : grammata ; deuxième mode : egrapse ; troisième : graphidi ; quatrième : technikos ; seconde figure, premier mode concluant : egrapse ; deuxième mode : kateche ; troisième



At this point, in order to support our reconstruction of the three lettered figures by the probable diagrams, we must recall at least the following passages from the *Prior Analytics*. Let us read these definitions of the three syllogistic figures in the light of the above diagrams.

“When three terms are so related to one another that the last is in the middle as a whole and the middle either is or is not in the first as a whole, it is necessary for thereto be a perfect syllogism of the extremes. I call ‘middle’ the term that is itself in another and in which there is also another - this is also middle by position (ὁ καὶ τῆ θέσει γίνεται μέσον)²⁷. The extremes are what is itself in another and in which there is another. For if A is predicated of every B and B of every Γ, it is necessary that A be predicated of every Γ” (*APr* I 4, 25 b 32-39).

“When the same [middle term] belongs to all of one and none of the other, or to all or none of both other terms, I call such a figure the second. In it, I call ‘middle’ the term that is predicated of both; and I call the extremes the terms of which it is predicated. The major extreme is the one lying next to the middle, the minor the one farther from the middle (μεῖζον δὲ ἄκρον τὸ πρὸς τῷ μέσῳ κείμενον· ἔλαττον δὲ τὸ πορρωτέρω τοῦ μέσου). The middle is posited outside the extremes and is first by position (τίθεται δὲ τὸ μέσον ἔξω μὲν τῶν ἄκρων, πρῶτον δὲ τῆ θέσει).²⁸

There will not be any perfect syllogism in this figure, but a syllogism will be possible, both if the terms are universal and if they are not. If they are universal, there will be

mode : metrion ; quatrième mode : acholon ; troisième figure, premier mode : hapasi, deuxième : sthenaros ; troisième : isakis ; quatrième : aspidi ; cinquième : homalos ; sixième : pheristo » (Cacourous’ translation).

²⁷ P.T. Geach in his translation of this text aptly states: “Clearly a reference to a diagram, now lost” (quoted in: Ackrill, 1987: 27).

²⁸ Once again rightly P. T. Geach: “This reference is not to logical relations of terms, but to their places in some diagram” (ibidem, p. 29). This was correctly noted by Alexander ad locum (in *APr.* 72, 11): Διὰ τῆς καταγραφῆς τῶν ὄρων καὶ τῆς τάξεως ἐδήλωσεν ἡμῖν, ὅτι τῆς μεζζονος προτάσεως τῆς ἐν τῷ πρώτῳ σχήματι ἀντιστροφείσης τὸ δεύτερον σχῆμα γέγονεν· ἡ γὰρ θέσις καὶ ἡ τάξις, ἦν εἶρηκε, τῶν ὄρων καὶ τὸ προτετάχθαι τὸν μέσον καὶ μετ’ αὐτὸν κείσθαι τὸν μεζζονα τὴν ἀντιστροφῆν ἐκείνης δηλοῖ τῆς προτάσεως.

“By the diagram of the terms and their order he has made clear to us that it is when the major premise in the first figure is converted that the second figure comes about. For the position and order of the terms which he describes — the fact that the middle is put first in order and the major supposed after it — make clear that it is the major premise which is converted.” (Barnes — Bobzien — Flannery — Ierodiakonou’s translation slightly modified). Alexander in his commentary (in *APr.* 78,4; 301, 9–19; 381, 8–12) made other references to the diagrams.

a syllogism whenever the middle belongs to all of one and none of the other, on which-ever term the privative is joined to, otherwise there will never be any syllogism" (*APr* I 5, 26 b 34-27 a 3).

„If one term belongs to all, and another to none of the same subject, or both to all or both to none, I call this a third figure. By the middle in it, I mean the term of which both are predicated, and by the extremes I mean the predicated terms. The major extreme is the one that is further from the middle, while the minor is the one that is the nearer (μειζον δ' ἄκρον τὸ πορρώτερον τοῦ μέσου, ἔλαττον δὲ τὸ ἐγγύτερον). The middle is posited outside the extremes and is last by position (τίθεται δὲ τὸ μέσον ἔξω μὲν τῶν ἄκρων, ἔσχατον δὲ τῇ θέσει). Now, no perfect syllogism will come about in this figure, but it will be possible both when the terms are universal in relation to the middle and when they are not" (*APr* I 6, 28 a 10-18).

In order to better understand the spatial and graphic arrangement of the three terms in our reconstruction of the three diagrammed figures, let us additionally quote the following two passages. Note that only in the first of these texts the middle term is marked by Γ and not by B (as above in our diagrams):

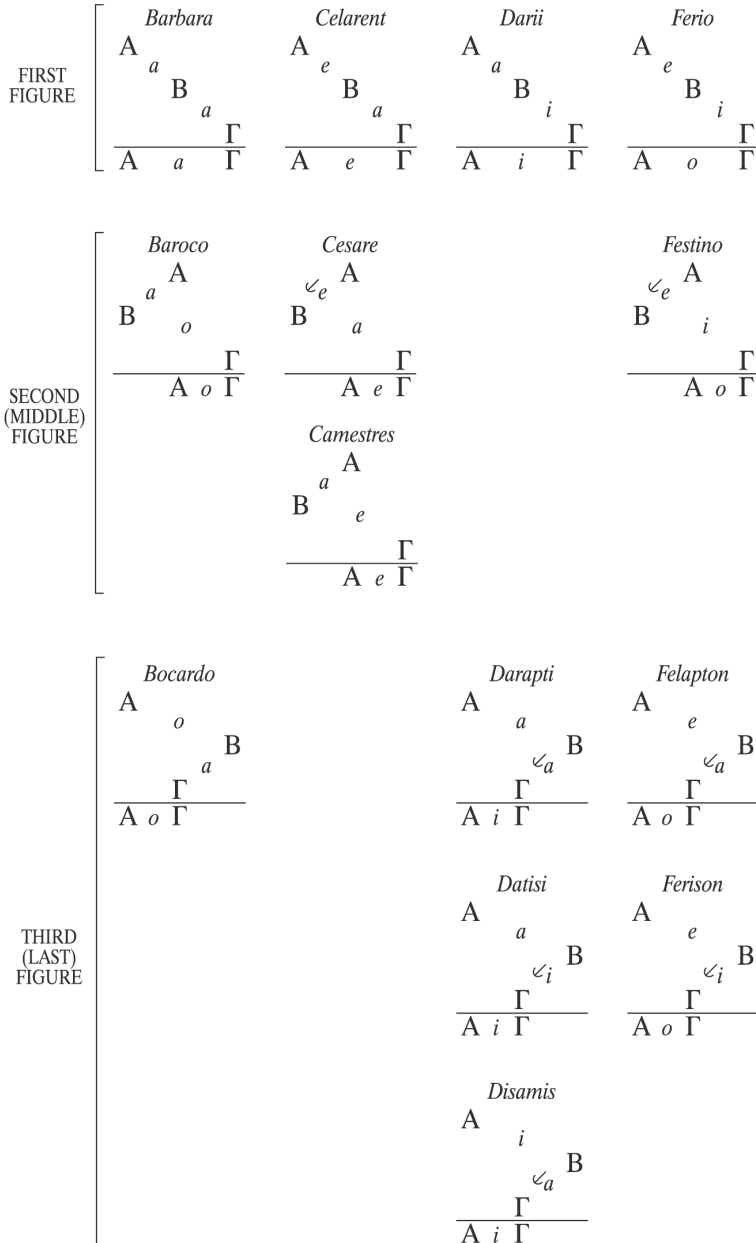
“If, then, it is necessary to take some common term in relation to both, and this is possible in three ways (for either by predicating A of Γ and Γ of B, or by predicating Γ of both A and B, or by predicating A and B of Γ), and these ways are the figures mentioned, then it is evident that every syllogism will necessarily come about through one of these figures. For the argument is the same if the connection to B is made through more than one term, for there will be the same figure also in the case of many terms. It is evident, then, that the ostensive syllogisms come to their conclusion in the aforementioned figures.” (*APr* I 23, 41 a 13-22).

“Now when the middle predicates and is predicated, or if it predicates and something else is denied of it, then there will be the first figure; when the middle predicates and is denied of something, there will be the middle figure; and when others [terms] are predicated of it, or the one is denied, the other predicated of it, there will be the last figure. For this was the position of the middle term in each figure (οὕτω γὰρ εἶχεν ἐν ἐκάστω σχήματι τὸ μέσον). The same holds also when the premises are not universal, for the determination of the middle remains the same (ὁ γὰρ αὐτὸς διορισμὸς τοῦ μέσου)” (*APr* I 32, 47 b 1-7).

Finally, in the third step of our reconstruction, let us display in the synoptic diagrams the formulae of three figures within their valid moods of analytical syllogisms (we adopt their medieval labels). For the sake of a uniform notation, we use a threefold array of letters A, B, Γ (as above), but now instead of arrows we put the fourth categorical schemes (*a, e, i, o*), and also separate the conclusions from the premises (by horizontal lines). Let us see inside these terms-letters all valid predications displaying those fourteen arguments in the figures. The rows and columns in these diagrams are designed to facilitate the synoptic account of all valid predicative interconnections, and also to analyze (loosen up) the imperfect syllogisms into the moods of the first figure, mainly by *antistrophe* of a *protasis*, when its predicate-subject order is convertible (see below by symbol \Leftarrow). It seems likely that Aristotle employed such configurations of blackboard diagrams, for

he called the second figure „the middle”, and the third figure — „the last” (the reader is referred to the diagrams on the following page).

FOURTEEN VALID MOODS ARRANGED INTO THREE FIGURES (A_{Pr}I 4–7)



Without going into details, we must note that from the unified and synoptical diagrams outlined above it is easy to see how the relevant syllogistic moods fit together within their analytical framework. There is special evidence for the superiority of the first figure within its analytical function in the syllogistic and apodeictic, and it is instructive to see how within other figures all valid predicative relations and transpositions can be accounted for, especially by conversion, but also by means of the two auxiliary methods: *reductio ad impossibile* and *ekthesis* (exposition). In only four moods of this figure (*Barbara*, *Celarent*, *Darii*, *Ferio*), the terms are related in such a manner that the transitivity of predications through the middle from the major to the minor becomes obvious at the very first glance. It is only in this figure that the perfect syllogism appears, without further terms from outside or any transformation among them, in order for the 'following of necessity' to be self-evident.²⁹

In many places of his *Analytics*, Aristotle speaks of these *schemata*, through (διὰ) or in (ἐν) which all syllogisms come about. These *schemata* refer primarily to their heuristic or analytical procedure and then, conversely, they are useful for the completion of syllogisms. These three figures are of great analytic importance not only because all categorical syllogisms are schematized and completed (τελειοῦσθαι) according to them, but principally because they are intended to provide the rules for the extended analysis of any given conclusion to be proven from the appropriate premises.

As a heuristic strategy, the analysis has a twofold sense: as a procedure of finding and completing the premises and as a procedure of reducing the syllogisms from one figure to another (*APr* 47 a 4; 49 a 19; 50 a 8; 50 a 17; *I* 45 *passim*). But Aristotle especially intends his analytical *schemata* to be of practical service. The following passages give a general recommendation of this analysis procedure:

"It is evident, then, that if the same term is not said several times in an argument, no syllogism will come about, for no middle term has been taken (οὐ γὰρ εἴληπται μέσον). Since we have seen what sort of problem a conclusion in each figure can be, whether universal or particular, it is evident that we not need to look for all the figures, but only for the one appropriate for each problem. And if it can be deduced in several figures, then we may recognize the figure by the position of the middle" (ὅσα δ' ἐν πλείοσι περαίνεται, τῇ τοῦ μέσου θέσει γνωριῶμεν τὸ σχῆμα) (*APr* I 32, 47 b 7-14).

„We must not overlook that not all conclusions in the same syllogism come through a single figure, but one is through this and one through another. It is clear, then, that we must also analyze them in this way (δῆλον οὖν ὅτι καὶ τὰς ἀναλύσεις οὕτω ποιητέον). And since not every problem occurs in every figure, but only certain ones in each, it is evident from the conclusion in which figure we should seek" (*APr* I 42, 50 a 5-10).

"We know that a syllogism does not come without a middle and that the middle is what is said several times. And the way one must watch out for the middle with relation

²⁹ The first figure is the most epistemonical (knowledge-giving). Aristotle's theory of apodeixis is based on the arguments in the first figures. „Finally, this figure has no need of the others; but they are thickened and increased through it until they come to the immediates (ἐτι τοῦτο μὲν ἐκείνων οὐδὲν προσδέεται, ἐκεῖνα δὲ διὰ τούτου καταπικνοῦται καὶ αὔξεται, ἕως ἂν εἰς τὰ ἄμεσα ἔλθῃ).” (*APo* I 14, 79 a 30-32).

to each type of conclusion is evident from knowledge of what sort of conclusion is proved in each figure” (*APr* II 19, 66 a 25-31).

Shortly then, in such an analytical procedure, the resolving of a problem (conclusion) consists in finding such predicative relations that connect the major with the minor by means of the middle one. The point of departure in the analysis is a given problem (conclusion), which is always known in advance, before the premises are decided on. In our notation, it is a substitution of one of the four forms of conclusion (namely: *A a I*; *A e I*; *A i I*; *A o I*). Accordingly, the very form of the conclusion suggests, by means of diagrams, the appropriate figures and moods into which it is to be resolved. The question lies in finding the position of the middle, and this allows us to recognize the figure, and — if the syllogism is „imperfect” — to reduce it to the first figure.

As we know, Aristotle in the *Prior Analytics* (I 8-22) develops with a much longer treatment his modal syllogisms within necessity and possibility of predication (belonging). This account involves also such an analytical reduction to the figures. The philosopher elucidates. “Each of the syllogism comes about in its own figure. [...] “It is now evident for this figure too when and how there will be a syllogism, in which cases it will be possible for the belonging and which cases for actual belonging. It is also clear that all these syllogisms are imperfect and that they are perfected through the first figure” (*APr.* I 22, 40 b 12-15 – Striker’s translation).

It would be instructive to survey them in the relevant diagrammatic configurations. It would (hopefully) throw some light on his modal logic, which is generally recognized to be confusing and unsatisfactory. But this complex and nowadays thoroughly discussed topic requires a separate treatment.³⁰

6. τὰ σχήματα τῶν κατηγοριῶν - τὰ σχήματα τῶν συλλογισμῶν – An unspecified and overlooked connection –

As we can see, Aristotle considers ‘following of necessity’ within the predicative transitivity and transparency owing to the middle term in the first figure to which the moods of the other two figures are reduced. In general, the three syllogistic figures constitute syntactical *schemata* for predicative inferences and for analytic reductions. But in Aristotle’s analytical approach there are more elementary ingredients that concern the syntax and semantics of the terms used in the syllogisms. Analogously to the *schemata* of syllo-

³⁰ For a new complex and highly technical approach to Aristotle’s modal syllogistic, see Patterson 1995; Nortman 1996; Thom 1996; Malink 2006; Rini 2011. M. Malink (2006) tried to disprove the opinio communis that Aristotle’s modal syllogistic is incomprehensible due to its many faults and inconsistencies. He gives a consistent formal model for it. Aristotelian modalities are to be understood as certain relations between terms as described in the theory of the predicables in the *Topics*. On the other hand, A. Rini (2011) provides a simple interpretation of Aristotle’s modal syllogistic using standard predicate logic. The result is an applied logic which provides the necessary links between Aristotle’s views of science and logical demonstration.

gisms, the philosopher conceived some syntactical and semantical *schemata* of categorical predication.

For Aristotle, to complete a syllogism is to take one term of another in order to connect the both extremes through the mediating term by virtue of its strong categorical predications. Let us recall the relevant passages:

“If one should have to complete a syllogism A of B, either as belonging or not belonging, it is necessary to take something of something (Εἰ δὴ δεῖοι τὸ Α κατὰ τοῦ Β συλλογισασθαι ἢ ὑπάρχον ἢ μὴ ὑπάρχον, ἀνάγκη λαβεῖν τι κατὰ τινος). [...]”

For, in general, we said that there will never be a syllogism for one term predicated of another unless some middle term has been taken which is related to each of the two somehow by categorical predications (μὴ ληφθέντος τινὸς μέσου, ὁ πρὸς ἑκάτερον ἔχει πως ταῖς κατηγορίας). [...]

So one must take a middle term for both which will connect the categorical predications, since a syllogism will be of this term in relation to that” (ὥστε ληπτέον τι μέσον ἀμφοῖν, ὁ συνάψει τὰς κατηγορίας, εἴπερ ἔσται τοῦδε πρὸς τόδε συλλογισμός — *APr* I 23, 40 b 31-32; 41 a 2-4; 11-13) Smith' translation slightly modified).

Hence, the middle term in syllogisms should link both extremes by strictly categorical predications. This is a prerequisite for any valid predications. The notion *kategoriai* (mentioned two times in the above passages) has been translated here as ‘categorical predications’. There can be no doubt that the *kategoriai* are basically ‘predications’, but they are also divided into ten ‘genera or schemata of categories (predicates)’: what-it-is, quantity, quality, relation, when, where, being placed, having, acting, being affected (*Top.* I 9). This is clear at least from the *Posterior Analytics* (I 22):

“Hence, when one term is predicated of another (ὅταν ἓν καθ’ ἑνὸς κατηγορηθῆ), either in what-it-is or as quality or quantity or relation or acting or being affected or where or when [...]. For of each there is predicated something that denotes (κατηγορεῖται ὁ ἄν σημαίνει) either a quality or a quantity or one of these [categories], or some other in the substance. But these are limited, and the genera of the predicates are limited (τὰ γένη τῶν κατηγοριῶν πεπεράνται) — either quality or quantity or relation or acting or being affected or where or when” (*APo.* 83 a 18-23; 83 b 13-17).

In this respect, the specification of the *gene ton kategorion* (cf. *Top.* 103 b 22) should not be interpreted as the highest genera, but strictly according to a different figure of predication (σχῆμα κατηγορίας — *Metaph.* 1024 b 12-15; cf. 1017 a 23; 1016 b 34; 1026 a 36; 1045 b 1-2; 1051 a 35; 1054 b 29; *Phys.* 227 b 4). Although it is crucial to properly understand what these *schemata* were for Aristotle's semantics of predication, the issue has for the most part been neglected.³¹ Without going into details, we must note that Aristotle's categories do not refer directly to the real things, but rather are concerned with the classification of things that are said and signified by the terms (subjects-predicates), whether according to the figures of predication these terms denote in categorical propositions a substance, a quality, a quantity etc. Strictly speaking, these figures specify some seman-

³¹ For the semantic interpretation of Aristotle's categories, see Wesoly 1984: 103–140; 2003: 11–35.

tic models for a correct predication of things (bodies) and their properties. In a certain sense, they were formal models, although Aristotle was obviously not familiar with our modern notion of formalization and, thereby, had no formal semantics. Thus, we claim that Aristotle's figures of categories have a great methodological importance for the semantics of predication and *eo ipso* for the syllogistic arguments.³²

In the *Prior Analytic* (36-37), we encounter two very concise, yet most instructive remarks on the semantics of the copula 'to be' and predication (belonging) that have as many meanings as the categories that have been distinguished.

ἀλλ' ὅσαχῶς τὸ εἶναι λέγεται καὶ τὸ ἀληθὲς εἰπεῖν αὐτὸ τοῦτο, τοσαυταχῶς οἴεσθαι χρῆ σημαίνειν καὶ τὸ ὑπάρχειν (*APr* 48 b 3-5).³³

"But in as many ways 'to be' is said and 'it is true to say' [means] the same, in so many ways one must think that denotes 'to belong'". —

Τὸ δ' ὑπάρχειν τόδε τῶδε καὶ τὸ ἀληθεύεσθαι τόδε κατὰ τοῦδε τοσαυταχῶς ληπτέον ὅσαχῶς αἱ κατηγορίαι διήρηνται, καὶ ταύτας ἢ πῆ ἢ ἀπλῶς, ἔτι ἢ ἀπλᾶς ἢ συμπλεγμένας· ὁμοίως δὲ καὶ τὸ μὴ ὑπάρχειν. ἐπισκεπτέον δὲ ταῦτα καὶ διοριστέον βέλτιον (*An. Pr.* A 37, 49 b 6-10).

"That 'this belongs to that' and that 'this is truly said of that ought to be taken in so many ways as the categories are divided, and these either in respect or *simpliciter*, and again either simple or compound. And similarly for 'not-belonging' as well. But these points must be better investigated and determined".

It is only regrettable that in Aristotle's preserved writings we do not find the above-promised clarification of the semantics of belonging, and that the scholars did not pay attention to this relevant issue.³⁴ However, from several other passages in Aristotle, we can reconstruct his semantics of categorical predication. In almost the same wording as above, Aristotle distinguishes the meanings of 'to be' according to the figures of categories in *Metaphysics* V 7.

καθ' αὐτὰ δὲ εἶναι λέγεται ὅσαπερ σημαίνει τὰ σχήματα τῆς κατηγορίας· ὅσαχῶς γὰρ λέγεται, τοσαυταχῶς τὸ εἶναι σημαίνει. ἐπεὶ οὖν τῶν κατηγορουμένων τὰ μὲν τί ἐστὶ σημαίνει, τὰ δὲ ποιόν, τὰ δὲ ποσόν, τὰ δὲ πρὸς τι, τὰ δὲ ποιεῖν ἢ πάσχειν, τὰ δὲ πού, τὰ δὲ ποτέ, ἐκάστῳ τούτων τὸ εἶναι ταῦτό σημαίνει·

"Those that are said 'to be' *per se* in as many ways as the figures of predication denote. For in as many ways [these] are said, in so many 'to be' [the figures] denote. Since, then, of predicates some denote a what-it-is, some quality, some quantity, some relative, some

³² At this point, one can hardly agree with de Rijk's major work (2002, vol. 1-2) that Aristotle's statement-making is copula-less, that the categories are 'appellations' ('nominations') and have nothing to do with the formula of predication. But these questions are in need of further examination, giving a new impetus to the study of Aristotle's syntax, semantic and analytics.

³³ Notice here Aristotle's adverbially wording: ὅσαχῶς (in as many ways...) — τοσαυταχῶς (in so many ways...) to express the analogy of meanings. Cf. *Metaph.* 1022 a 11.

³⁴ Bocheński (1951: 34) seems to be the only one to have appreciated this important topic: „Consequently the classification is not only one of the objects, but above all one of the modes of predication; and in the light of this we must note as historically false the widespread opinion accrediting Aristotle with the knowledge of only one type of sentence, that of class-inclusion”.

doing or being affected, some where, some when, so each of these predicates as the same 'to be' denotes" (*Metaph.* 1017 a 23).

These figures of categories specify the semantic models of predication which are used for valid predications in the subject-predicate propositions. Nonetheless, such a use of these figures of categories by Aristotle has not yet been fully recognized and investigated.

Admittedly, Aristotle coined up the same label *schemata* for the 'figures of syllogisms' and for 'figures of predication', but the structural analogies between them have not yet been properly acknowledged and explicated. As far as we know, nobody even attempted to integrate Aristotle's views on both these figures. It seems that the heuristic analysis by means of the syllogistic figures rests upon the semantic models of predication (belonging).

If this proposal is correct, Aristotle's analytics provides the 'terms' logic within the framework of categorical predications and analytical reductions. At its core, two formal *schemata* are to be found and these (i.e., the semantic figures of predications and the syntactical figures of syllogism) seem to be coordinated with each other through the pivotal function of the middle term.

7. πεπαιδευμένος τῶν ἀναλυτικῶν...

We can fully appreciate how important for Aristotle were his laborious findings in the analytics from his frequent references to the need of educating in this respect. In the polemical context, he speaks often about the lack of education in the analytics (ἀπαιδευσία τῶν ἀναλυτικῶν — *Metaph.* 1005 b 2-5; cf. *Metaph.* 995 a 13-14; *Metaph.* 1006 a 7; *EE* I 6, 1217 a 6-10); *EN* I 3, 1094 b 24-25). This requirement was related properly to the method of predication (belonging) and *eo ipso* to the demonstration (or scientific explanation) with its degree of exactness, and, thus, to the competence (παιδεία) resulting from the *Prior* and *Posterior Analytics*.

In this respect, the methodological comments at the beginning of the *De partibus animalium* prove very revealing. Aristotle distinguishes here two kinds of competence relevant to any given inquiry: the first-order education is understanding of the subject-matter, and the second-order education is judging the method of predication and explanation when it is made:

„For it is characteristic of an educated man to be able to judge aptly what is right or wrong in an exposition (τὸ δύνασθαι κρίναι εὐστόχως τί καλῶς ἢ μὴ καλῶς ἀποδίδωσιν ὁ λέγων) [...].

Hence, it is clear that in the inquiry into nature, too, certain terms must belong, such that by referring to them one will admit the manner of things demonstrated (δεῖ τινὰς ὑπάρχειν ὄρους τοιούτους πρὸς οὓς ἀναφέρων ἀποδέξεται τὸν τρόπον τῶν δεικνυμένων), apart from how the truth [of belonging] has it, whether thus or otherwise (χωρὶς τοῦ πῶς ἔχει τἀληθές, εἴτε οὕτως εἴτε ἄλλως)” (*PA* 639 a 5-18).

This remarkable passage is notoriously misunderstood by almost all translators and commentators.³⁵ In this context, the verb ὑπάρχειν can only refer to the „belonging” of some „terms” according to the relevant *schemata* of categorical predication. In this respect, we must properly invoke at least two of Aristotle’s methodological suggestions quoted above (*APr.* I 30; 37); namely, how to discern the terms of belonging, and what does mean ‘to belong’ in as many ways as the categories are divided, and as synonymous to the sense „this is truly said of that”. In general, the philosopher regarded the *paideia* in analytics as a logical or methodological competence concerning the predication, arguments and the way of explanation.

8. Analytical *σχήματα* vs. dialectical *τόποι*

In the *Prior Analytics*, the most technical and inventive treatise of Aristotle, the philosopher was able to expound a unified and coherent method within the *genesis* and reduction of the syllogisms to the figures. For Aristotle, it was the only one and a truly unique strategy for creating syllogisms through the terms that follow and are followed. He proves particularly convinced of this in the following passage:

„It is evident from what has been said, then, not only that it is possible for all syllogisms to come about through this method, but also that this is impossible through any other. For every syllogism has been proved to come about through some of the aforementioned figures, and these cannot be constructed through other terms than those that follow and those that are followed by each term (διὰ τῶν ἐπομένων καὶ οἷς ἔπεται ἕκαστον). For the premises and the taking of the middle is from these (αἱ προτάσεις καὶ ἡ τοῦ μέσου λήψις), so that there cannot even be a syllogism through other terms” (*APr* I 29, 45 b 36-46 a 2).

But Aristotle’s belief here was all too optimistic, as his claim about the universal application of this analytical method to resolve the dialectical and rhetorical arguments turned out to be unsuccessful or inapplicable in the *Topics* and the *Rhetoric*. Indeed, already in the *Prior Analytics* (I 44), the philosopher was well aware of the fact that the hypothetical and dialectical syllogisms cannot be reduced to those diagrammed figures. This is a complex and separate issue that here cannot be pursued further (See Striker 2008: 235-239).

Nevertheless, Aristotle in the *Topics* did not find a unique universal method for resolving the dialectical arguments, since he assumed that there was a plurality of methods or tools (ὄργανα), of which the most important are clearly the *topoi* (cf. *Top.* I 6, 102 b 35-103 a 1; cf. I 18, 108 b 32-33). However, Aristotle’s *Analytics* and *Topics* consider analytically the reasoning as finding the premises from a given conclusion. The starting point in dialectic is a given *protasis* that seeks something acceptable (ἔνδοξον), and a given problem that

³⁵ As far as I know, only M. Schramm (1962: 152–153) considers the issue in the same way as we do. “Allerdings sollte man ὅροι nicht im Sinn von Definitionen pressen; ὅρος kann insbesondere für den Syllogismus, einen logischen Term bezeichnen und entspricht in dieser Funktion der des Begriffs. Der weitere Fortgang des Abschnittes bietet keine Definitionen, sondern methodische Vorschriften, deren Inhalt, in Form von Termen gefaßt, den Übergang von der Regel zur kanonischen Anwendung vermitteln würde.”

is subject to speculation (θεώρημα). Thus, the dialectical *protaseis* and *problemata* fall into the so-called predicables (*definition, unique property, genus, and accident*) which are further specified according to the genera of the categories (predications) and which are also very important for the specification of the *topoi* (lit. 'places').³⁶

"These, then, are the tools by means of which the syllogisms are made. The *topoi* against which the aforementioned are useful are as follows." (*Top.* I 18, 108 b 32-33).

However, Aristotle did not express the acceptable premises by means of the lettered terms in the function of the middle, mainly because in the dialectical syllogism there is no middle term, and such a reduction to the diagrammed figures is, thereby, out of the question. However, with reference to the *endoxa* Aristotle instructs us how to provide them by using some diagrammatic collections (διαγραφαί):

„We should construct tables (τὰς δὲ διαγραφὰς ποιῆσθαι), setting them down separately about each genus, for example about the good or animal, and about every good, beginning with what it is (ἀπὸ τοῦ τί ἐστίν)" (*Top.* I 14, 105 b 12-15).

In the *Rhetoric*, Aristotle states that „the same thing is an element and a topos (στοιχεῖον καὶ τόπος); for an element or a topos [is a schema] under which many enthymemes fall (ἔστιν γὰρ στοιχεῖον καὶ τόπος εἰς ὃ πολλὰ ἐνθυμήματα ἐμπίπτει). [...] but these things are the subject of syllogisms and enthymemes" (*Rhet.* II 26, 1403 a 17-23).

It is important to note that Aristotle evidently borrowed his rhetorical notion of *topos* from Greek geometry.³⁷ Indeed, the *topoi* as *elements* seem here to be analogical to the analytical meanings of the *elements* as constituents of diagrams and of demonstrations. Admittedly, there is a structural analogy between the reduction into the *schemata* in the analytics and the reduction into the *topoi* in the dialectic and rhetoric. Thus, the dialectic *topoi* and the analytical *schemata* serve some heuristic rules or tools (ὄργανα) for argumentation, respectively.

9. πόρισμα (*corollarium*)

Nowadays, when we investigate the syllogistic and the diagrams, we come to think of yet another famous proposals of the diagrams, i.e., the ones by Euler and Venn. However, these accounts are quite distant from the ancient and medieval ones, as they concern the tracing of syllogistic validity, but no longer have any strict connection with Aristotle's analytics and the relevant reduction to the figures. For this reason we omit them here. At any rate, they clearly testify to the ingenuity and vitality of the Aristotelian syllogistic.³⁸

³⁶ For a discussion of the difference between the *Analytics* and the *Topics* see Smith 1997 and Slomkowski 1997.

³⁷ This was recently well shown by Eide 1995: 5–21. "Aristotle's phrase suggests that many enthymemes 'fall into' a certain type or pattern determined by the topos, just as lines in a diagram 'fall into' certain places determined by the geometrical locus." (Eide 1995: 12).

³⁸ We admire G. Englebretsen's significant contributions to the linear diagrams for syllogisms and also his illuminating rediscovery of Aristotle's logic (see the bibliography).

By focusing on the textual evidence, we have tried to offer a more clear and coherent reconstruction of Aristotle's lost syllogistic figures. Many questions outlined above need further consideration. Some ideas of Aristotle's analytics that for a long time have been regarded as idiosyncratic can now gain validity and sound logical sense, despite their traditional and modern criticism. When seen in its original context, the "analysis concerning the figures" appears to be much more relevant than it is frequently thought. Accordingly, Aristotle's logical and methodological achievements demand not only a new historical reading, but also a modern recognition that will do justice to its uniqueness and specificity.

So far the significance of the three figures as the syntactic framework of analytics seems to have escaped even some of the most renowned historians of Aristotle's logic. From a modern point of view, the division of syllogisms into figures seems to be of no real importance. Łukasiewicz (1957: 23) believes that "the division of the syllogism into the figures has only a practical aim: we want to be sure that no true syllogistic mood is omitted". Somewhat differently, Smith (1994, 135) speaks of the 'figured argument' in lieu of the 'syllogism' and of the 'theory of the figures' instead of the 'syllogistic'. In this view, Aristotle's claim that *every deduction can be reduced to a figured argument or a series of such arguments* is false from the standpoint of modern logic.

Although Aristotle's analytics may *prima facie* seem irrelevant to contemporary logical theories, this does not necessarily mean that it lacks inventiveness, substantive validity and inner coherence. Our concern here has been to appropriately recognize the meaning and the specificity of the Stagirite's analytical framework. There can be no doubt that these *schemata* were of great importance for Aristotle's syllogistic account. Hence, our goal has been merely to do justice to them in this respect.

It is common knowledge that Aristotle's syllogistic understood as a term calculus has been accused of being limited and inadequate as a tool for the formalization of mathematical demonstrations. This is true to some extent, but in the light of the most recent research, we can attempt to better tackle the question how Aristotle's syllogistic could be made adequate (sufficient) to represent Greek mathematical proofs (cf. H. Mendell, 1998). It has to be borne in mind that Greek mathematicians proved their theorems with lines through constructible figures, whereas modern mathematicians and logicians prove these through axioms. Similarly, in geometry the diagrammatic approach was substituted by the algebraic one, while in logic, the analytical approach was replaced by the formal and symbolic one. Modern mathematics and logicians renounced the notation typical of Greek mentality for the sake of the formalized and axiomatic paradigms.

Nonetheless, we accept the recent view of Aristotle's syllogistic as a natural, non-axiomatic, deductive system that dealt with the predications involving relations between the terms (Ebbinghaus, 1964; Corcoran, 1974). We can *mutatis mutandis* say that his logical system was similar to what we nowadays call 'predicate' or 'term' logic.

Moreover, the syllogistic of the *Analytics* as an epistemic metascience is not oriented ontically (in the sense of class inclusion) but rather epistemically, i.e., it serves as a formal model for the apodeictic of the *Posterior Analytics*. In such an analytical-

-deductive system, Aristotle's concern was not formalism in itself, but rather a heuristic approach for starting with any given 'problem' and finding the premises to it. As far as the methodological aspect of the *episteme* is concerned, it has to be stressed that Aristotle very penetratingly elaborated this theoretical framework in the *Posterior Analytics*, whose general concern is the epistemonical syllogism or scientific explanation.

The most remarkable feature of Aristotle's analytics as a whole is undoubtedly its ingenuity, thoroughness and perspicacity. He was well aware of the great difficulties and his own contribution into the field. That is why he encouraged other researchers to show indulgence for the deficiencies of his method, and to be at the same time most grateful for his discoveries (cf. *SE* 34). We believe that many things remain yet to be discovered in studying Aristotle, things that he himself could not have foreseen.

Contemporary logicians seem to have no patience and cognitive curiosity for Aristotle's analytics, as they rashly neglect or belittle the importance of his formulation of the analytical figures. The point of view of modern mathematical formal logic is obviously instructive and illuminating, but if it is quite differently-oriented, and, therefore, it may sometimes prevent us from obtaining a historically adequate interpretation of Aristotle's achievements.

It has been over fifteen years, since I proposed a reconstruction of Aristotle's diagrams of the syllogistic figures. My proposal seems to have provoked little feedback or stimulating discussions, as only two eminent scholars have provided me with certain critical remarks: Prof. George Englebretsen (Canada) and Prof. Jacques Brunschwig (France). Hereby, I would like to take this opportunity and thank them wholeheartedly for their inspiring criticisms. I hope that this new and significantly revised version of the previous suggestion will provoke more discussions. Moreover, I would like to thank Dr Mikołaj Domaradzki for the inspiring suggestions, encouragement and assistance with the English translation of this article. At the same time, I obviously acknowledge that all mistakes and infelicities are mine alone.

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ΑΝΑΛΥΣΙΣ ΠΕΡΙ ΤΑ ΣΧΗΜΑΤΑ. Restoring Aristotle's Lost Diagrams of the Syllogistic Figures

The article examines the relevance of Aristotle's analysis that concerns the syllogistic figures. On the assumption that Aristotle's analytics was inspired by the method of geometric analysis, we show how Aristotle used the three terms (letters), when he formulated the three syllogistic figures. So far it has not been appropriately recognized that the three terms — the major, the middle and the minor one — were viewed by Aristotle syntactically and predicatively in the form of diagrams. Many scholars have misunderstood Aristotle in that in the second and third figure the middle term is outside and that in the second figure the major term is next to the middle one, whereas in the third figure it is further from it. By means of diagrams, we have elucidated how this perfectly

accords with Aristotle's planar and graphic arrangement.

In the light of these diagrams, one can appropriately capture the definition of syllogism as a predicative set of terms. Irrespective of the tricky question concerning the abbreviations that Aristotle himself used with reference to these types of predication, the reconstructed figures allow us better to comprehend the reductions of syllogism to the first figure. We assume that the figures of syllogism are analogous to the figures of categorical predication, i.e., they are specific syntactic and semantic models. Aristotle demanded certain logical and methodological competence within analytics, which reflects his great commitment and contribution to the field.

KEYWORDS

Aristotle, analysis, analytics, syllogistic figures, diagrammatic notation